Modal analysis and intensity of acoustic radiation of the kettledrum

Lamberto Tronchin[a)]
DIENCA—CIARM, University of Bologna, Viale Risorgimento, 2, I-40136 Bologna, Italy

(Received 17 May 2004; revised 5 October 2004; accepted 12 October 2004)

The acoustical features of kettledrums have been analyzed by means of modal analysis and acoustic radiation (p/v ratio) measurements. Modal analysis of two different kettledrums was undertaken, exciting the system both by a hammer and a shaker. Up to 15 vibrational modes were clearly identified. Acoustic radiation was studied using two ways. Based on previous experiments of other researchers, a new parameter, called *intensity of acoustic radiation (IAR)*, has been defined and measured. Results show a strict relationship between IAR and the frequency response function (FRF, which is the v/F ratio), and IAR also strongly relates the modal pattern to acoustic radiation. Finally, IAR is proposed for vibro-acoustical characterization of kettledrums and other musical instruments such as strings, pianos, and harpsichords.
[DOI: 10.1121/1.1828552]

PACS numbers: 43.75.Yy, 43.75.Hi, 43.40.Dx [NHF] Pages: 1–8

I. INTRODUCTION

Modal analysis and acoustic radiation in musical acoustics are widely used to analyze vibro-acoustical behavior of musical instruments. Many studies have been conducted characterizing the acoustic radiation of many instruments using these techniques. Since sound radiation is strongly related to modal patterns, a correlation between resonance frequencies in vibrating structures and sound production should exist. On the other hand, previous studies find a negative correlation between acoustic radiation and frequency response function (FRF, which is the v/F ratio) of membranes or plates in instruments such as piano and harpsichord (Suzuki, 1986; Giordano, 1998). It could be argued that pianos and harpsichords are quite complicated instruments, and so understanding their sound radiation could be hampered by their complex structure. Therefore, the study on frequency response, modal analysis, and acoustic radiation will be undertaken for two tympani, where sound generation is largely from the membrane, so the correlation between FRF and sound radiation should be found easily. The comparison between experimental modal patterns and previously published results could suggest the most appropriate measurement technique for characterizing vibro-acoustical properties of musical instruments.

A new vibro-acoustical parameter, able to properly relate sound production and FRF, is required especially for tympani, where sound generation and modal analysis are strongly related. Applications of this extend beyond musical acoustics into the modeling of musical instruments in auditoria.

II. TYMPANI PHYSICS: MODAL ANALYSIS

Most drums normally produce a sound of indefinite pitch, with the notable exceptions of tympani and tablas, the former of which is used in the orchestra (Rayleigh, 1945). A drum skin or membrane is distinctively different from a string in that the string vibrations follow a harmonic series, while the membrane vibrates in more complex ways. A circular membrane's nodes form concentric circular lines, and straight lines following the membrane's diameters. Each partial contributing to the instrument's sound corresponds to a particular vibration mode. In Fig. 1, the first 12 modes are represented, with the ratio between the frequency of the corresponding partial and the mode (0,1).

How can a kettledrum produce a note of defined pitch? Previously, physicists (Rayleigh, Benade, Rossing) have studied the frequency ratio between the fundamental and upper modes, and the effect of air-loading. Rayleigh noted that the pitch corresponds to (1,1) mode. Benade (1990) measured the first ten components of the sound of a 25-in. kettledrum tuned on C (130.8 Hz). Rossing and his staff (in several studies from 1976 to 1998) (Rossing, 1976, 1977, 1982) investigated vibration modes of kettledrums, finding that all the vibration modes with only diametric nodes are in harmonic ratios to each other. Furthermore, the vibration modes (1,1), (2,1), (3,1), (4,1), and (5,1) are respectively in frequency ratios of 1, 1.5, 2, 2.44 (about 2.5), and 2.90 (about 3) with the mode (1,1) (Fletcher and Rossing, 1998). These modes are almost in harmonic ratios to a missing fundamental one octave below the (1,1) mode. Although this missing fundamental could be rebuilt by the human ear, normally this does not happen, perhaps because the intensity and duration of the harmonics are insufficient to enable the ear to grasp the harmonic spectrum.

Rossing found that air-loading is important in tuning the kettledrum's partials. Air-loading is the effect of the mass of air in the vicinity of the membrane, which lowers the natural frequencies of vibration from those which would occur in a vacuum. This effect is strongest for low frequencies, and influences especially the (1,1) mode. Additionally, the kettle affects the modes, increasing the circular mode frequencies and reducing the diametric mode frequencies (Tubis and

[a)]Electronic mail: tronchin@ciarm.ing.unibo.it

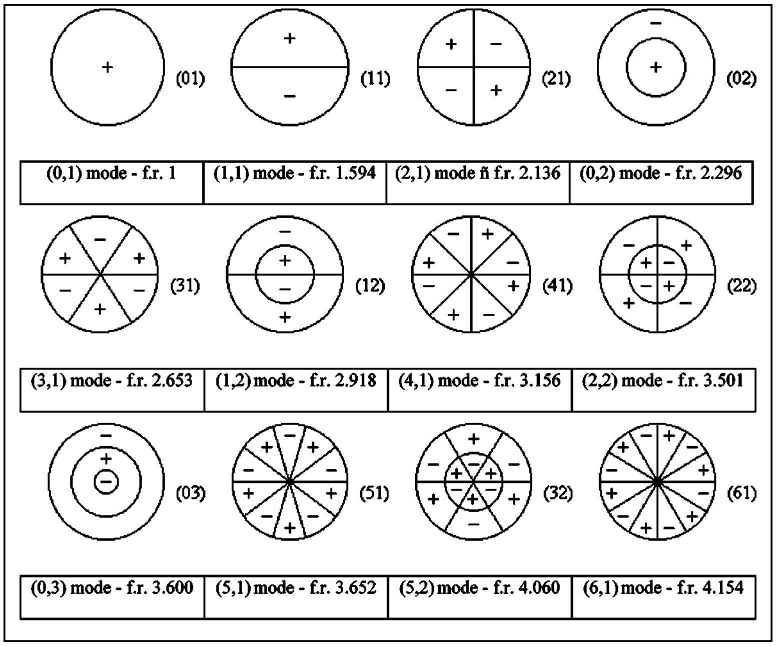

FIG. 1. The ratio (f.r.) between the frequency of the corresponding overtone and that of the ‘‘fundamental’’ mode (0,1) is signed under each mode. This is not a harmonic sequence.

Davis, 1986). This affect is most important for the (0,1) mode. Many efforts have been made in determining a theoretical model of air-loading, with perhaps the best known model described by Christian *et al.* (1984).

Two other effects, namely the air modes in the kettle and the bending stiffness of the membrane, play a minor role compared to the effect of the weight of the air, and contribute to the ‘‘fine tuning’’ of the membrane frequencies. Although the air in the kettle has its own modes of resonance which could potentially interact with the membrane modes, Rossing found a great difference in the frequencies of membrane modes and kettle air modes. Consequently, kettle air vibrations have a very weak interaction with membrane vibrations. The bending stiffness of the membrane could raise the frequencies of higher partials (a similar effect is found in piano strings), but this effect is negligible for the lower partials, which are of most interest in understanding the vibroacoustical properties of the instrument.

The radiation of sound from a baffled kettledrum membrane depends on the vibrating mode. In the (0,1) mode, the membrane acts as a monopole source, whereas in the (1,1) mode it acts as a dipole source, and so on (Fletcher and Rossing, 1998). The directivity of sound radiation is fairly noticeable in an anechoic room, but not so in a normal (slightly reverberant) room. Consequently, measurements of acoustic radiation should not be preformed locating the microphone at the center of the membrane, but one-fourth from the edge of the kettle. Spatial averaging of directivity patterns can be achieved by using many striking points around the circle representing normal positions for striking the instrument in musical performance.

III. ACOUSTIC RADIATION

The efficiency of acoustic radiation is a measure of the effectiveness of a vibrating surface in generating sound power. It could be defined by the relationship:

$$\sigma = \frac{W}{\rho_0 c S \langle \overline{v_n^2} \rangle} \tag{1}$$

in which W is the sound power radiated by a surface with area S, which could be obtained by integrating the far-field intensity over a hemispherical surface centered on the panel, and $\langle \overline{v_n^2} \rangle$ is the space-averaged value of the time-averaged normal distribution of velocity (Fahy, 1989).

From this general definition various measurement methods useful for the study of sound emission could be obtained.

A. Research on acoustic radiation

Previous studies on this argument have been conducted on the soundboards of the piano and of the harpsichord. K. Wogram, H. Suzuki, and N. Giordano studied the soundboard of the piano using different measurement methods.

Wogram (1984) used the parameter F/v, defining F as the excitation force and v as the resulting velocity at the point of excitation. He reported that it exhibits a maximum at a frequency near or below 1 kHz, and that it falls sharply below 100 Hz, and above 1 kHz. He found that it falls typically by a factor of 10 as the frequency is varied from 1 to 5 kHz.

Suzuki (1986) used the ‘‘surface-intensity method,’’ defined as

$$I = \mathrm{Re}[p(\alpha/j\omega^*)/2], \tag{2}$$

where I is the average intensity in time, perpendicular to the vibrating surface, measured in near field (about 30 cm from the radiating surface), ω is the angular frequency, Re and* are the real part and the complex conjugate of a complex number, p and α are the pressure and the normal acceleration at the measuring point, and $j = (-1)^{1/2}$. His study was conducted over a limited frequency range, but imply that the integrated sound intensity normalized by the input power is approximately constant from 200 Hz to 5 kHz.

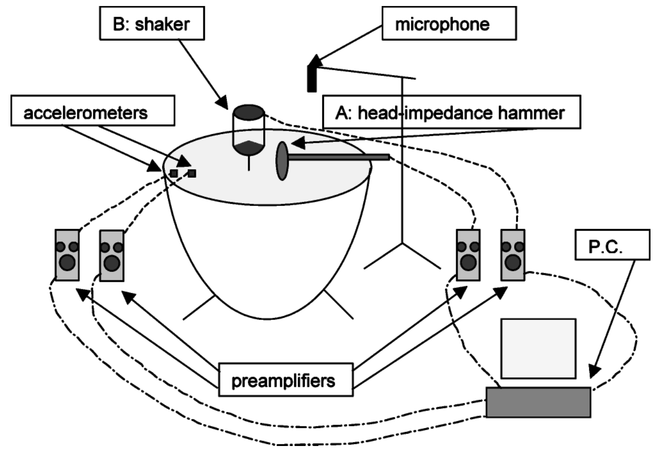

FIG. 2. Schematic diagram with all the experimental apparatus.

Giordano (1998) used the parameter p/v, where p is the sound pressure measured in near field and v is the velocity of the soundboard. In all the measured points p/v is greatest at about 1 kHz, and it falls off below a few hundred hertz and above 5 kHz.

Suzuki's and Giordano's results agree, but appear to be in contrast with Wogram's results on the average value of F/v, which falls by a factor of 10 or more from 1 to 5 kHz. Is important to notice that all of these studies have one result in common: the resonance frequencies did not coincide with those of acoustic emission; on the contrary they often had negative correlation.

IV. EXPERIMENT 1: MODAL ANALYSIS

Vibration and acoustic measurements were conducted at the same time. The FRF (v/F) of the instruments was obtained from the vibration measurements. Up to 15 modes were studied, in order to check the results with previous research. Two tympani with different features were analyzed—these were made available by the Conservatorium of Cesena in Italy. Both tympani were in normal condition, with used membranes. The first was a plexi-glass Adam 25-in. (about 65 cm) kettledrum with a Remo mylar skin and a central reinforce, tuned to approximately 166 Hz (corresponding to E). The second was a copper 25-in. (65 cm) Ludwig kettledrum with a mylar skin and no central reinforce, tuned to approximately 145 Hz (corresponding to D). Figure 2 is a schematic diagram of the experimental apparatus.

A. Experimental configuration

The membrane was excited by percussive impulses in 213 measuring points, fixed on a square grid with 4-cm grid intervals. Measurements were conducted at the same points on the two instruments. Waveforms were recorded and stored on a personal computer, and waveform analysis was conducted using Aurora software (Farina, 1997).

1. Adams kettledrum

The following instrumentation was used:

(i) Hammer Brüel & Kjær Type 8203,
(ii) Accelerometer Brüel & Kjær Type 4374,

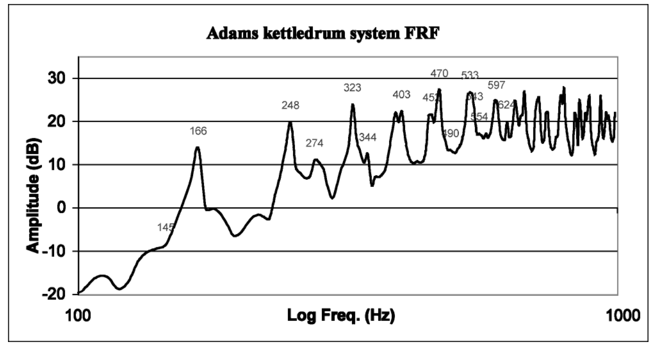

FIG. 3. Adams kettledrum system FRF: excitation induced by the hammer.

(iii) two charge amplifiers Brüel & Kjær Type 2635, and
(iv) PC equipped with 20-bit A/D converter, 96-kHz sample rate sound-board.

Based on reciprocity theory, the accelerometer was located at a fixed point on the membrane, being a point usually hit by the performer (10 cm from the edge of the skin), while the tympanum was excited by the hammer at 213 positions. In order to minimize measurement error, an average of ten consecutive impacts for each measurement position was used.

The amplitude of space-averaged FRF measured for Adam tympanum is presented in Fig. 3.

2. Ludwig kettledrum

The following instrumentation has been utilized:

(i) two accelerometers Brüel & Kjær Type 4374,
(ii) two charge amplifiers Brüel & Kjær Type 2635,
(iii) Electrodynamic Mini-Shaker Brüel & Kjær Type 4810, and
(iv) PC equipped with 20-bit A/D converter, 96-kHz sample rate sound-board.

In this case (Fig. 4) the vibration of the structure was measured by placing the accelerometer at each of the 213 fixed points and exciting the system through the electrodynamic shaker placed in the normal striking point for the player (which was the same point as for the Adams instrument). The shaker provides an alternative way to excite the structure, which theoretically should enhance the quality of the mea-

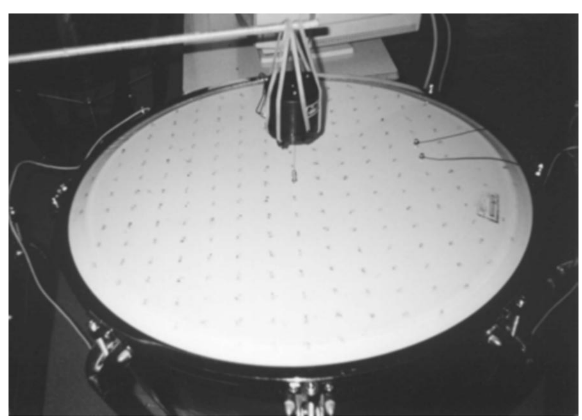

FIG. 4. Ludwig kettledrum: shaker placed in the middle.

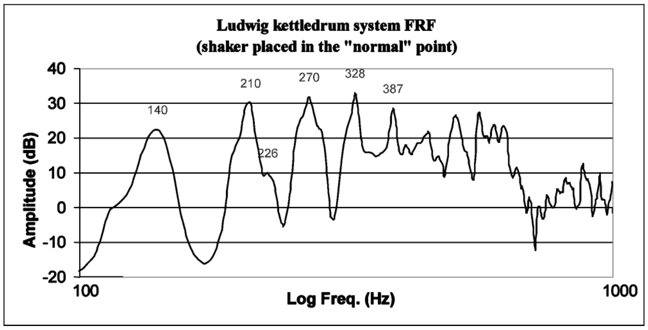

FIG. 5. Ludwig kettledrum system FRF: shaker placed in the normal striking point.

sured FRF, especially at mid-high frequencies.

In order to obtain impulse responses (IRs) between the shaker and each accelerometer position a logarithmic, swept sine waveform 10 s in duration ranging from 20 Hz to 20 kHz was employed. In a second step the IRs were transformed to the frequency domain and the final FRF (amplitude and phase) obtained from the average of all 213 single FRFs. Finally, the mappings of the vibrational modes were obtained in the same way as for the Adams instrument. Another set of measurements was conducted applying the shaker in the middle of the membrane, and testing the influence of the shaker position on the measurement of FRF (Fig. 4). The amplitudes of the two space-averaged FRFs obtained positioning the shaker in the two positions on the membrane are presented in Figs. 5 and 6, respectively.

B. Results of the modal analysis

1. Adams kettledrum

For the Adams kettledrum, 15 vibration modes in the frequency range from 140 to 540 Hz were found. In Table I the results are summarized and compared to Rossing's results.

The results were very similar to those expected. Circular and mixed modes corresponded almost perfectly to those found by Rossing. The frequencies corresponding to diametrical modes were lower than expected. The discrepancy grew with increasing modal frequency. The (5,1) mode, which is not very harmonic in theory, was the least similar to Rossing's measurements; a very small discrepancy was found for the modes (2,1), (3,1), (4,1). The mode (5,1) and the mode (6,1) had frequencies lower than those of the

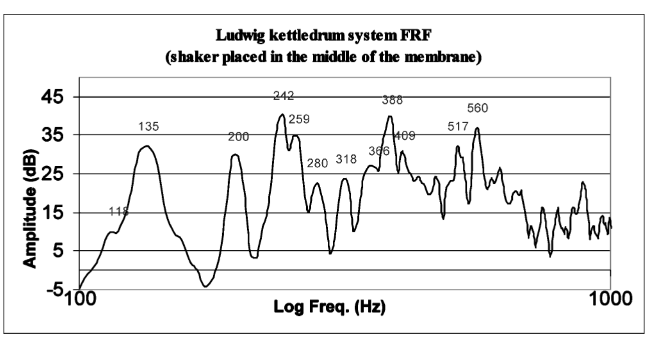

FIG. 6. Ludwig kettledrum system FRF: shaker placed in the middle of the membrane.

TABLE I. Frequencies and measured ratios and comparison to Rossing's results. In gray: frequencies that help the harmonicity of tympani.

Modes m, n	Frequencies $f_{m,n}$ (Hz)	Ratios $f_{m,n}/f_{(11)}$	Ratios found by Rossing: Measured	Ratios found by Rossing: Calculated
(0,1)	145	0.87	0.81	0.80
(1,1)	*166*	*1.00*	*1.00*	*1.00*
(2,1)	*248*	*1.495*	*1.50*	*1.52*
(0,2)	274	1.65	1.65	1.68
(3,1)	*323*	*1.95*	*1.97*	*2.00*
(1,2)	344	2.07	2.00	2.27
(4,1)	*403*	*2.43*	*2.44*	*2.48*
(2,2)	452	2.72	2.86	2.74
(5,1)	*470*	*2.83*	*2.91*	*2.94*
(0,3)	490	2.95	2.71	2.97
(6,1)	533	3.22	⋯	3.40
(3,2)	543	3.27	⋯	3.29
(1,3)	554	3.34	⋯	⋯
(7,1)	597	3.59	⋯	⋯
(4,2)	624	3.76	⋯	⋯

modes (0,3) and (3,2), whereas previous studies found the reverse. Circular and mixed modes corresponded perfectly to theory (Fig. 7).

Two similar mappings, one corresponding to the mode (5,1), the other not corresponding to any resonance peak, were found. The second one had the same features of the mode (5,1), but it was in a harmonic ratio with the mode (1,1). It could be interesting to investigate on the origins of this vibration mode.

2. Ludwig kettledrum

Only three vibration mode mappings are reported (Fig. 8), with a lower definition. In this case the use of shaker and accelerometers, throughout the membrane, could have increased the mass-loading effect. Modal frequencies were shifted some hertz higher, especially in the low frequency range.

The FRF graphics showed a peak at almost 3 kHz, probably attributable to the resonance frequency of the very thin bar that connected the shaker to the membrane.

Comparing the FRF obtained by exciting the system at the normal striking position to the FRF obtained by exciting the middle of the membrane, the second one has more stimulated resonance peaks corresponding to the circular and mixed vibration modes.

A comparison with the Adams kettledrum reveals that the head-impedance hammer method gives more precise space averaged frequency responses than the shaker method. This means that the first method is especially suitable when the main purpose of the measurements is to obtain modal shapes (at reasonably low resonance frequencies). Moreover, using reciprocity theory, the measurements are usually very quick, and in a short time a mesh of hundreds of points can be measured. The shaker method gives more precise frequency responses than hammer, even though at very high frequencies harmonic artifacts could occur due to additional mass loading, steel-stick resonance, and wax-damping. However, in both cases the FRFs obtained are suitable for experiments investigating acoustic radiation.

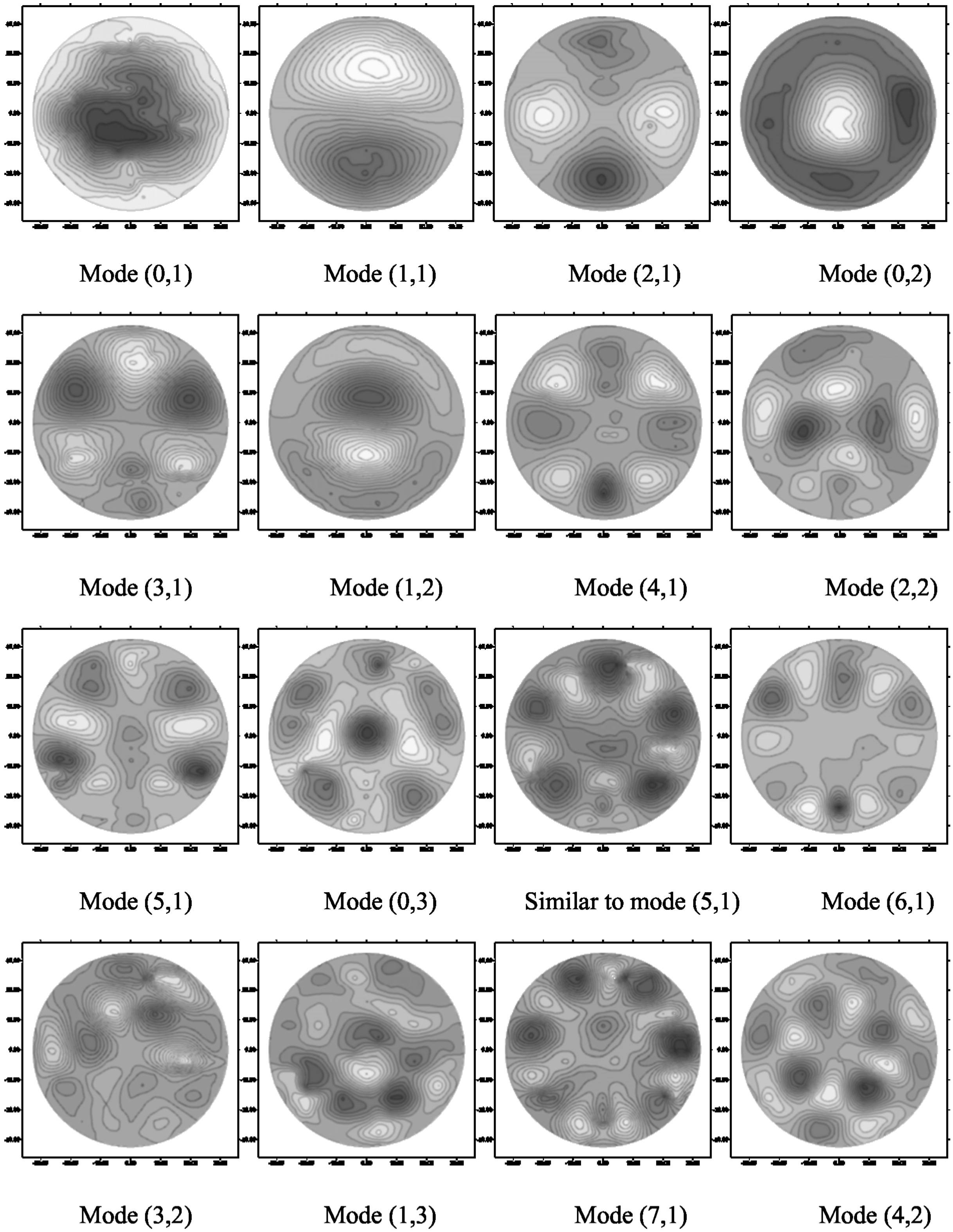

FIG. 7. Mappings of Adams kettledrum vibrational modes.

V. EXPERIMENT 2: ACOUSTIC RADIATION

The second part of the investigation was dedicated to the measurement of acoustic radiation. The same musical instruments used in modal analysis were studied.

A. Definitions and experimental configuration

Acoustic radiation was measured with the Adams kettledrum used in the modal analysis, but tuned on 145 Hz, with the same 213 striking points used for the modal analysis.

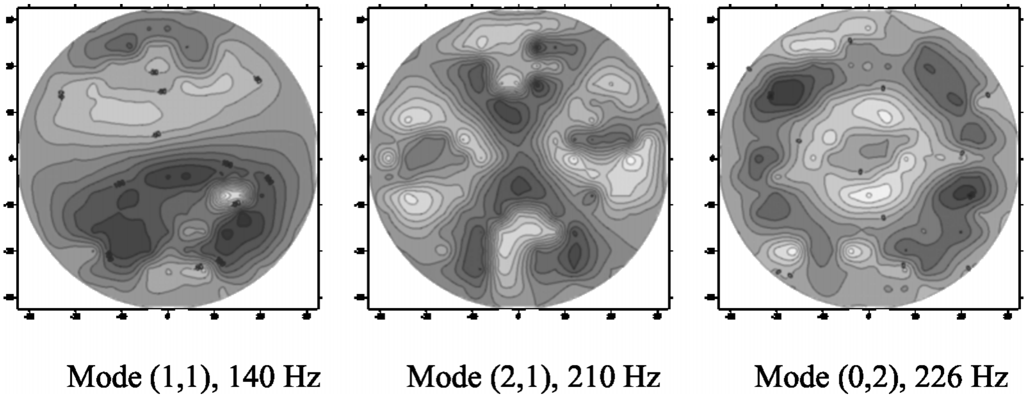

FIG. 8. Mappings of Ludwig kettledrum vibrational modes.

Two different parameters of the acoustic radiation were measured and compared: the complex ratio p/v, which is the parameter used by Giordano, and a new parameter that can be defined as *intensity of acoustic radiation* (IAR), because it is a parameter between acoustic intensity and radiation.

IAR is defined as the space-averaged amplitude of cross spectrum between sound pressure caused by the movement of the vibrating surface (the membrane) and the velocity of the vibration of the membrane itself. An omnidirectional microphone was located in a fixed position at about 25 cm over the membrane, one-fourth from the edge of the kettle, and the accelerometer was mounted at the same points utilized during modal analysis. The measurements were repeated for each position of the accelerometer, avoiding errors caused by sound directivity.

The measurements were conducted in a slightly reverberant room, where reverberation time helps to average radiation of sound caused by (0,1) and (1,1) modes. At higher frequencies the room acoustics did not influence the measurements. Moreover, the space-averaging of the data conducted by moving the transducers thorough the membrane enhanced the measurements.

Sound pressure p was measured in near field, at 25 cm from the membrane, as previously reported by Suzuki and Giordano (Figs. 2 and 9). In order to properly measure radiation of sound, the distance between the radiating surface and the microphone should be one-fourth of the wavelength, and hence 25 cm was considered a good compromise for low and high frequencies. However, further study would be useful to optimize the microphone position during such measurements, balancing the need to avoid excessive reverberation from the room and the need to achieve sufficiently uniform radiation from the membrane.

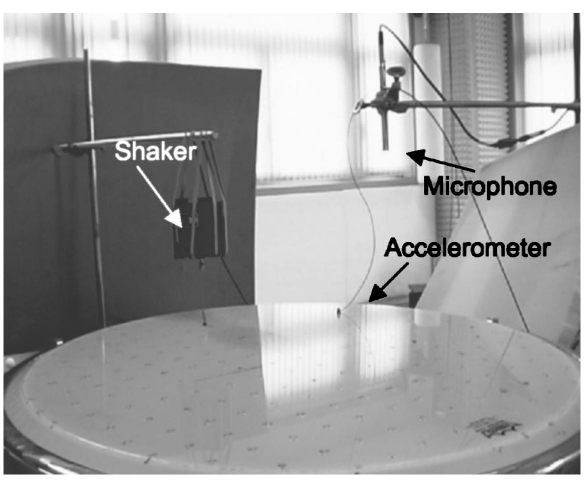

FIG. 9. Adams kettledrum: acoustic radiation measurements.

The following instrumentation was used:

(i) accelerometer Brüel & Kjær Type 4374,
(ii) charge amplifier Brüel & Kjær Type 2635,
(iii) condenser microphone connected to sound level meter Larson Davies LD model 2900B,
(iv) electrodynamic Mini-Shaker Brüel & Kjær Type 4810, and
(v) PC equipped with 20-bit A/D converter, 96-kHz sample rate sound-board.

The measurements were conducted in the same way as described in Sec. IV A 2. The IRs, measured at the microphone and the accelerometer, were simultaneously measured by means of a logarithmic sine sweep generated by the shaker (Farina, 1997). In a second step, the synchronous p and v IRs were postprocessed, and, in the frequency domain, the amplitude and phase of the transfer function p/v and cross spectrum $p \cdot v$ were calculated.

B. Measurements of acoustic radiation: Results and comments

The efficiency of acoustic radiation (p/v) appears approximately constant from 270 to 3800 Hz, with a peak in this range at 1200 Hz (Fig. 10). At a finer scale, rapid fluctuations of p/v can be observed. Even though a kettledrum is quite different from a piano or harpsichord, the curve is still opposite in phase with the FRF, meaning that the p/v curve has minima at the same frequencies where the FRF has maxima—therefore it has the same features as those of Suzuki and Giordano.

Results for $IAR\,(p \cdot v)$ show a different pattern to the FRF or p/v results (Fig. 11). The frequency range of maximum sound radiation intensity is between 140 and 900 Hz, with a progressive small decrease in amplitude as the frequency grows. Tall peaks and valleys characterize this zone. The strongest intensity was found in the frequency range of the mode (1,1), at 156 Hz, but the amplitudes of the modes

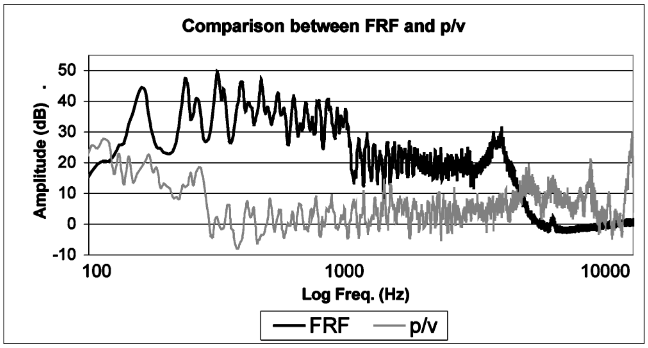

FIG. 10. Results of the measurement of the acoustic radiation: comparison between FRF and p/v.

(2,1) and (0,2), respectively, at 226 and 247 Hz, were substantial, too. Above 900 Hz, amplitudes decrease suddenly, remaining constant from 900 to 3000 Hz. Substantial peaks and valleys are recorded also from 900 to 1500 Hz, whereas from 1500 to 3000 Hz they become decreasingly evident. All the graphics obtained exciting the system through the shaker have a peak around 3000–3200 Hz that could not be related to the sound properties of tympani, but could correspond to the resonance frequency of the very thin bar connecting the membrane with the shaker.

The most important results of the investigation derive from the comparison between the FRFs and the parameters p/v and $IAR\,(p\cdot v)$, respectively. In the first case, frequencies with great radiation efficiency (p/v) do not correspond to the resonance frequencies of the frequency response (to the vibration modes), and indeed are in antithesis to them. The curve corresponding to p/v is in phase opposition to that of the FRF curve, following previous studies of Suzuki and Giordano (Fig. 10). This should not be a surprising result. The definition of p/v is close to the mechanical impedance, and therefore it would explain sound losses rather than sound generation of the soundboard. Besides, FRF is strongly related to sound generation, and this should apply to sound generation of vibrating surfaces, and therefore sound intensity.

The comparison between the graphic of the FRF and the graphic of the intensity of the acoustic radiation $IAR\,(p\cdot v)$ shows interesting results (Fig. 11). The graphics has a very similar curve and resonance frequencies correspond perfectly to sound emission frequencies. The surprising correlation between FRF and IAR suggests the adoption of the new parameter as descriptor of acoustic radiation. Moreover, the space-averaged amplitude of cross spectrum $p\cdot v$ between the sound pressure and velocity of the membrane, measured over a large number of points, suggests calling the new parameter *intensity* of acoustic radiation, since it is related to radiation (velocity v on the membrane), and the relation is defined like the sound intensity product, as described in Sec. V A.

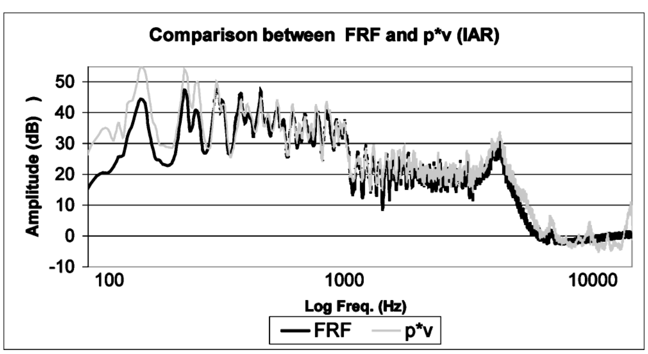

FIG. 11. Results of the measurement of the acoustic radiation: comparison between FRF and $p\cdot v$.

VI. CONCLUSIONS

Acoustic radiation measurements and modal analysis were conducted in two different kettledrums, namely Adams and Ludwig models. The measurements were conducted as indicated in previous papers. Two procedures for exciting the membrane were used and compared: the head-impedance hammer and the shaker. The FRFs obtained with the two techniques for each kettledrum were quite similar, whereas modal shapes were better using the hammer. The shaker gave better frequency results up to 3 kHz, but the resonance of the bar connecting the shaker to the membrane was found at about 3 kHz. The mappings of the 15 individual vibration modes were very clear, and frequency ratios agreed approximately with the theoretical ones. A high degree of correspondence was obtained for the circular and mixed vibration modes, whereas the diametric modes yielded frequencies slightly lower than the theoretical ones.

Acoustic radiation was measured in two different ways. In the first case the complex ratio (p/v) between sound pressure and the vibration velocity of the membrane was calculated. This is the method used by Giordano. In the second case the space-averaged amplitude of cross spectrum $(p\cdot v)$ between sound pressure, measured at a fixed point at 25 cm far from the instrument, and the vibration velocity of the membrane measured at more than 200 points was calculated. This is a new parameter called *intensity of acoustic radiation (IAR)*. Comparing the graphics of FRF and p/v, it can be observed how the resonance frequencies are often in opposition to those of acoustic emission, in accordance with previous studies conducted on soundboards of the piano. Applying IAR, the resonance frequencies correspond perfectly to those of sound emission, and the curves of the two graphics are very similar. The IAR parameter is well related to frequency response function and for this reason is preferred to p/v. It is a medium parameter between acoustic intensity and acoustic radiation, and so is suitable to measure the sound generating characteristics of musical instruments with vibrating soundboards. This parameter can be used to qualify and define the directivity of musical instruments, which is important for architectural acoustics, as well as for auralization processes.

ACKNOWLEDGMENTS

The author wishes to thank Alessio Buttazzoni, Alessandro Cocchi, and Angelo Farina for their help during the measurements and calculation of physical parameters on the two kettledrums. Furthermore, he wishes to thank Densil Cabrera for his help on the English proof of this work.

Benade, A. H. (**1990**). *Fundamental of Musical Acoustics*, 2nd ed. (Dover, New York).

Christian, R. S., Davies, R. E., Tubis, A., Anderson, C. A., Mills, R. I., and Rossing, T. D. (**1984**). ''Effects of air loading on tympani membrane vibrations,'' J. Acoust. Soc. Am. **76**, 1336–1345.

Fahy, F. J. (**1989**). *Sound Intensity* (Elsevier Applied Science, London).

Farina, A., and Righini, F. (**1997**). ''Software implementation of an MLS analyzer, with tools for convolution, auralization and inverse filtering,'' preprints of the 103rd AES Convention, New York, 26–29 September.

Fletcher, N. H., and Rossing, T. D. (**1998**). *The Physics of Musical Instruments*, 2nd ed. (Springer Verlag, New York).

Giordano, N. (**1998**). ''Sound production by a vibrating piano soundboard: Experiment,'' J. Acoust. Soc. Am. **104**, 1648–1653.

Rayleigh, J. W. (**1945**). *The Theory of Sound* (Dover, New York), Vol. 1.

Rossing, T. D. (**1976**). ''Acoustics of percussion instruments—Part I,'' Phys. Teach. **14**, 546–556.

Rossing, T. D. (**1977**). ''Acoustics of percussion instruments—Part II,'' Phys. Teach. **15**, 172–178.

Rossing, T. D. (**1982**). ''The physics of kettledrums,'' Sci. Am. **247**(5), 172–178.

Suzuki, H. (**1986**). ''Vibration and sound radiation of a piano soundboard,'' J. Acoust. Soc. Am. **80**, 1573–1582.

Tubis, A., and Davis, R. E. (**1986**). ''Kettle-shape dependence of timpani normal modes,'' paper K2-7, 12th International Congress on Acoustics, Toronto.

Wogram, K. (**1984**). ''Akustische Untersuchungen an Klavieren,'' in *Der Piano—und Flÿgelbau* (Verlag Das Musikinstrument, Frankfurt); English version, *Acoustical Research on Pianos: Vibrational Characteristics of the Soundboard*, Das Musikinstrument.